Estimation Problems

> **Note:** Remember, Fermi estimations are designed to be imprecise enough to be able to calculate in your head. Your answers may be slightly different!

1. A ream of paper (500 sheets) is 5.4cm thick. Estimate the number of sheets needed to reach the height of an average man.

2. There are 3 rats for every human in London. How many rats are there in London?

3. How many golf balls would fit in a suitcase?

4. How many steps would it take for a human to travel around the world once?

5. How many times greater is the price of a small shopping centre than the price of a business degree from a red-brick university?

6. Find the number of London double-decker buses that would have the same mass as the Moon.

Estimation Problems Answers

1. A ream of paper (500 sheets) is 5.4cm thick. Estimate the number of sheets needed to reach the height of an average man.

   Estimate the height of a ream of paper to be 0.05m.

   Estimate the height of a man to be 1.8m.

   The number of reams needed is 1.8 ÷ 0.05 = 36 reams.

   500 × 36 = 18 000 sheets

2. There are 3 rats for every human in London. How many rats are there in London?

   Estimate the population of London to be 10^7

   The number of rats is therefore 3×10^7

3. How many golf balls would fit in a suitcase?

   Estimate the volume of a golf ball to be 10^2 in cm^3

   Estimate the volume of a suitcase to be $10^2 \times 10^2 \times 5 \times 10 = 5 \times 10^5$ in cm^3

   $(5 \times 10^5) \div 10^2 = 5 \times 10^3$

   5000 golf balls.

4. How many steps would it take for a human to travel around the world once?

   Estimate the stride of a human to be 1m.

   Estimate the circumference of the Earth to be 4×10^7 metres.

   $\frac{4 \times 10^7}{1} = 4 \times 10^7$

   = 40 000 000 steps

5. How many times greater is the price of a small shopping centre than the price of a business degree from a red-brick university?

   Estimate the price of a shopping centre to be 3×10^8 in pounds.

   Estimate the price of a business degree to be 3×10^4 in pounds (3 years at £10 000 per year).

   $\frac{3 \times 10^8}{3 \times 10^4} = 10^4 = 1000$

6. Find the number of London double-decker buses that would have the same mass as the Moon.

   Estimate the mass of a bus to be 10^4 in kg.

   Estimate the mass of the Moon to be 10^{23} in kg.

Rounding and Financial Formulae

Simple interest = $P \times r \times t$

where P = principal, r = interest rate multiplier and t = time period in years.

Compound interest = $P(1 + r)^t$

where P = principal, r = interest rate multiplier, t = time period in years

Cash flow = **Income – Expenses**

Leverage ratio = $\frac{\text{total liabilities + total debts}}{\text{total income}}$

Rule of 72 = $\frac{72}{r}$ (years)

where r = interest rate (not a multiplier)

1. When Pal last checked, she had £2000 in her bank account, correct to 1 significant figure. Her bank gives 3% simple interest (correct to the nearest whole number). Calculate the upper bound of interest earned after 3 years.

2. Lily bought a car costing £9500, correct to the nearest £100. It depreciates in value at a compound rate of 6.2% per year (correct to 1 decimal place). Calculate the lower bound of the value of the car after 5 years.

3. Ali wants to work out his cash flow. He knows his income is roughly £3200 a month (correct to 2 significant figures) and his expenses are around £2860 correct to the nearest £10. Calculate the upper bound of his monthly cash flow.

4. The **rule of 72** is a way of estimating how long an investment will take to double. Raj has invested some money at a 3% interest rate, correct to the nearest whole number. Estimate the lower bound of the amount of time it will take for his investment to double.

5. A leverage ratio is a way of calculating the ability of a company to meet its financial obligations. For anyone who is thinking of lending to the company, a figure of 0.5 or lower is ideal.

    The total liabilities of a company are £10 million, correct to the nearest million, its debts are valued at approximately £4 million, correct to 1 significant figure, and its income is £28 million, correct to the nearest million. Calculate the upper and lower bounds of this company's leverage ratio. Hence, write down whether you think lending to this company would be a good idea.

Rounding and Financial Formulae Answers

1. When Pal last checked, she had £2000 in her bank account, correct to 1 significant figure. Her bank gives 3% simple interest (correct to the nearest whole number). Calculate the upper bound of interest earned after 3 years.

   Simple interest = 2500 × 0.035 × 3 = £262.50

2. Lily bought a car costing £9500, correct to the nearest £100. It depreciates in value at a compound rate of 6.2% per year (correct to 1 decimal place). Calculate the lower bound of the value of the car after 5 years.

   Compound interest = 9450 × 0.9375^5 = £6843.66 (to 2d.p.)

3. Ali wants to work out his cash flow. He knows his income is roughly £3200 a month (correct to 2 significant figures) and his expenses are around £2860 correct to the nearest £10. Calculate the upper bound of his monthly cash flow.

   Cash flow = 3250 – 2855 = £395

4. The **rule of 72** is a way of estimating how long an investment will take to double. Raj has invested some money at a 3% interest rate, correct to the nearest whole number. Estimate the lower bound of the amount of time it will take for his investment to double.

   Rule of 72 = $\frac{72}{2.5}$ = 28.8 years

5. A leverage ratio is a way of calculating the ability of a company to meet its financial obligations. For anyone who is thinking of lending to the company, a figure of 0.5 or lower is ideal.

   The total liabilities of a company are £10 million, correct to the nearest million, its debts are valued at approximately £4 million, correct to 1 significant figure, and its income is £28 million, correct to the nearest million. Calculate the upper and lower bounds of this company's leverage ratio. Hence, write down whether you think lending to this company would be a good idea.

   Upper bound = $\frac{10\ 500\ 000 + 4\ 500\ 000}{27\ 500\ 000}$ = 0.545 (to 3s.f.)

   Lower bound = $\frac{9\ 500\ 000 + 3\ 500\ 000}{28\ 500\ 000}$ = 0.456 (to 3s.f.)

   The final answer will be student dependant. You might say that it is roughly around 0.5, however if you choose to consider the upper bound you might not decide to lend to this company.